



# Modeling of static and dynamic behavior of 2.9 THz Quantum Cascade Lasers.

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pour Systèmes  
Embarqués\*

**Abstract :** Quantum Cascade Laser are at the same point of development nowadays than were the first semiconductor lasers in the 1960's. Indeed it still needs to be cooled at cryogenic temperatures (between 4K and 70K for our 2.9 THz QCL, even though some could be used at room temperature with decreased output power

## Rate equations of a Quantum Cascade Laser

$$\frac{\partial N_3}{\partial t} = \eta \cdot \frac{I}{q} - \frac{N_3}{\tau_3} - G \cdot (N_3 - N_2) \cdot S$$

$$\frac{\partial N_2}{\partial t} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} + G \cdot (N_3 - N_2) \cdot S$$

$$\frac{\partial N_1}{\partial t} = \frac{N_3}{\tau_{31}} + \frac{N_2}{\tau_{21}} - \frac{N_1}{\tau_{out}}$$

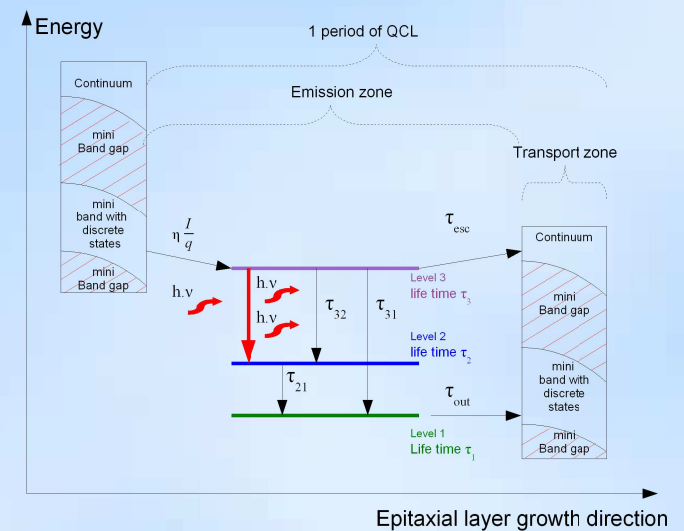
$$\frac{\partial S}{\partial t} = N_p \cdot G \cdot (N_3 - N_2) \cdot S - \frac{S}{\tau_s} + \beta \cdot \frac{N_3}{\tau_{sp}}$$

## Static behavior

- P(I) curves at different temperatures
- Threshold current versus temperature variations
- Comparison with experimental curves

## Dynamic behavior

- Small-signal frequency response
- Transfer function
- Bandwidth evaluation
- Variations of the transfer function with bias intensity and temperature



Optical gain  $G$  :  $G = \Gamma \cdot \sigma \cdot \frac{c}{n}$

- with :
- Optical confinement  $\Gamma = 0.27$
  - Optical linear gain  $\sigma = 1.6 \cdot 10^{-9} \text{ cm}^2$
  - Light velocity  $c = 3 \cdot 10^8 \text{ m/s}$
  - Refractive index  $n = 3.3$

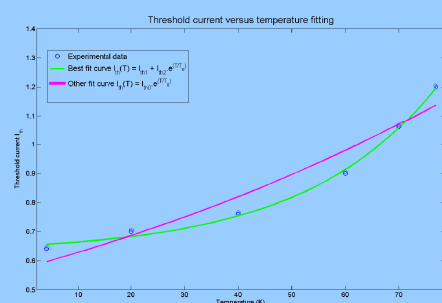
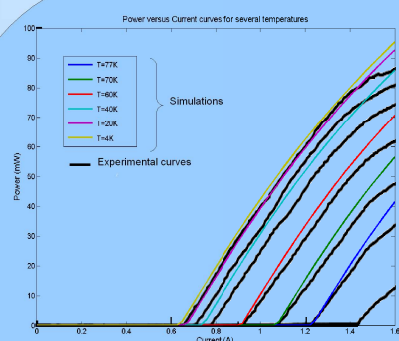
Invert of photon life time :  $\frac{1}{\tau_s} = \alpha \cdot \frac{c}{n} = (\alpha_i + \alpha_m) \cdot \frac{c}{n}$

- with :
- mirror losses  $\alpha_m = \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right)$
  - internal losses  $\alpha_i = 20 \text{ cm}^{-1}$

## Others life times :

- $\tau_{21} = 0.3 \text{ ps}$
- $\tau_{31} = 1.3 \text{ ps}$
- $\tau_{32} = 2.4 \text{ ps}$
- $\tau_{sp} = 7.0 \text{ ns}$
- $\tau_{out} = 0.5 \text{ ps}$

- Spontaneous Emission factor  $\beta = 1 \cdot 10^{-4}$   
Current injection efficiency  $\eta = 0.9$   
Number of periods  $N_p = 90$   
Cavity length  $L = 3 \text{ mm}$   
Cavity width  $w = 200 \mu\text{m}$



In order to fit the variations of the threshold current with temperature, it is common to use the expression :

$$I_{th}(T) = I_{th1} + I_{th2} \cdot \exp\left(\frac{T}{T_0}\right)$$

It performs much better result than the well known law :

$$I_{th}(T) = I_{th0} \cdot \exp\left(\frac{T}{T_0}\right)$$

With the first relation, Matlab can optimize the different coefficients so that we obtain :

$$I_{th1} \approx 0.63 \text{ A} \quad I_{th2} \approx 25 \text{ mA} \quad T_0 \approx 25 \text{ K}$$

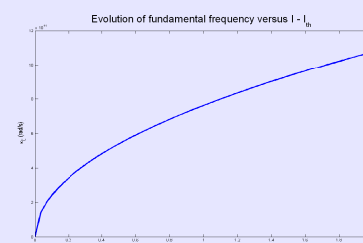
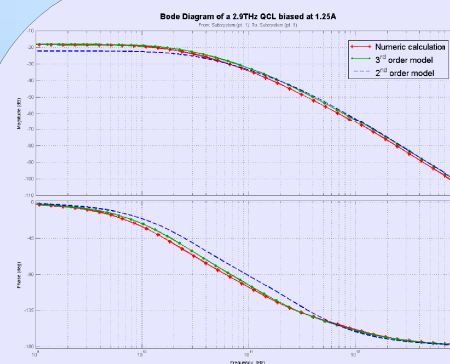
These values are in great agreement with some we can read in articles

The threshold current can be related to the optical gain as

$$I_{th} = \frac{\alpha}{N_p \cdot g} \quad \text{with} \quad g = \tau_3 \left( 1 - \frac{\tau_{21}}{\tau_{32}} \right) \cdot \frac{1}{q \cdot n} \cdot G$$

That leads to :

$$G(T) = \frac{G_0}{1 + \gamma \cdot \exp\left(\frac{T}{T_0}\right)} \quad \text{with} \quad G_0 = \Gamma \cdot \sigma \cdot \frac{c}{n} \quad \text{and} \quad \gamma \approx 0.04$$

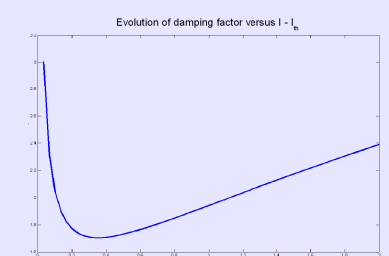


$$H(p) = \frac{H_0}{1 + 2 \frac{\xi}{\omega_0} p + \left( \frac{p}{\omega_0} \right)^2}$$

with  $\omega_0 = \sqrt{\frac{2 \cdot G \cdot S_0}{\tau_s}}$

leads to  $f_{3dB} \approx \frac{\sqrt{3} \cdot \omega_0}{2\pi} \approx 160 \text{ GHz}$

→ Great potential for free space high speed digital transmission



Modeling of this 2.9THz QCL by a non-resonant 2<sup>nd</sup> order transfer function system seems to lead to good results, in agreement with other research. Evolution of damping factor with bias current show that this QCL is always non-resonant

$\omega_0$  increases as the square root of the bias current, result well known for Fabry Perot semiconductor edge emitting laser.

